ON SUPERSTABLE FIELDS WITH AUTOMORPHISMS

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The Lie-Kolchin theorem states, essentially, that every connected solvable algebraic group over an algebraically closed field has a nilpotent derived group. This was generalized by Zil'ber and Nesin (independently) to groups of finite Morley rank. It was known that all ingredients of Nesin's proof generalize easily to superstable groups (satisfying an appropriate connectedness condition), except for the non-existence of definable groups of automorphisms of the field. The purpose of this note is to prove this fact: if F is a field, G a group of automorphisms of F, and (F,G) is superstable, then G = (1).

All groups are taken to be ∞-definable in $\mathbb{C}$, the universal domain of a superstable theory. We will use the notation of [M], and the theory of local weight in groups from [H, §3.3]. The basic definition is that of a regular type. The idea is that the elements of the group are co-ordinatized by n-tuples of realizations of the regular type. Thus for example if $A = (Z/2Z)^{\omega}$, with generic type p, then $B = A \times A$ is p-simple: an element of B is a pair of elements of A. But if $B = (Z/4Z)^{\omega}$, and A is identified with 2B, then B is not p-simple: an element of B can be analyzed in terms of p, but not in one step. Call a group p-connected if it is p-simple, connected, and has a generic type domination-equivalent to a power of p. One has the following existence property.

Fact 1. Let G be a group, H a group acting on G, and suppose the generic type of G is non-orthogonal to the regular type p. Then G has a normal, H-invariant subgroup N such that G/N is p-simple.