MODEL THEORETIC VERSIONS OF WEIL'S THEOREM ON PREGROUPS

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In 1955, A. Weil published a paper ["On algebraic groups of transformations", Am. J. of Math., vol.77 (1955), p:355–391] where, starting from a variety V over some algebraically closed field K, together with a binary operation on V which has "good" properties (associativity, rationality) on a large piece of V (generic points), he constructs an algebraic group G over K, whose multiplication is an extension of the given one on generic points and which is birationally equivalent to V.

More precisely:

Let K be an algebraically closed field and let V be an irreducible variety over K such that there is a mapping f: VxV → V with the following properties:

(i) if a,b are independent generic points of V over K, then

$$K(a,b) = K(a,c) = K(b,c)$$

(ii) if a,b,c are independent generic points of V over K, then

$$f(f(a,b),c) = f(a,f(b,c)).$$

Then there is an algebraic group G over K which is birationally equivalent to V, such that this birational equivalence takes f(a,b), for a,b independent generics of V, to the product of the images of a and b.

Model–theorists working on stable groups became interested in this theorem in the following context: first, recall that, by a stable (ω–stable) group, we mean a group (G,·) definable in M^n for M a model of a stable (ω–stable) theory or interpretable, i.e. definable on some quotient of M^n by some definable equivalence relation.

Amongst the first natural examples of ω–stable groups are algebraic groups over an algebraically closed field K (they are definable in the theory of K in the language of fields).