NON-ASSOCIATIVE RINGS OF FINITE MORLEY RANK

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INTRODUCTION

Stable associative rings have been investigated by Cherlin–Reineke [Ch–Re], by Baldwin–Rose [B–Ro] and by Felgner [Fe] too early in the history of stable algebraic structures to get the attention they deserve. Rose started to investigate stable non–associative rings [Ro] in the late 1970's but again his work did not get the attention of model theorists. Macintyre's classification of ω_1–categorical fields [Mac2] and its generalization to superstable fields [Ch–S] and to ω–stable division rings [Ch2] became important because, we think, of their importance in the study of stable groups. We believe in the near future stable rings (associative or not) and their stable modules will become an important research area in applied model theory. They arise naturally in the study of stable groups. Here we list 4 instances:

a) Zil'ber classified ω_1 – categorical associative rings of characteristic 0 as indecomposable algebras over an algebraically closed field of characteristic 0 [Zi2]. He announces in [Zi3] that the same methods classify also ω_1 – categorical nilpotent Lie algebras over $\mathbb{Q}$ and that using Campbell–Baker–Hausdorff formula (see e.g. [Jac 1]), one can deduce that ω_1 – categorical torsion–free nilpotent groups are algebraic groups over an algebraically closed field of characteristic 0.

b) Let R be an arbitrary ring (not necessarily associative, does not necessarily have a unit). Then on the set $G = R \times R$ we can define a group multiplication by

$$(x,y)\,(x_1,y_1) = (x + x_1,\, y + y_1 + xx_1).$$

This group can also be viewed as

$$G = \left\{ \begin{pmatrix} 1 & x & y \\ 0 & 1 & x \\ 0 & 0 & 1 \end{pmatrix} : x, y \in R \right\}$$

with the obvious multiplication. It is easily checked that G is nilpotent of class ≤ 2 and is Abelian iff R is commutative. Since G is interpretable in R, it inherits

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