

GENERIC FORMULAS AND TYPES A LA HODGES

by

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§0. Introduction.

Let T be a complete theory with monster model \mathbb{C} and A a subset of \mathbb{C} . Certain complete types $p \in S(\mathbb{C})$ have the "privilege" of being non-forking over A . The smaller A is, the harder it is not to fork over it. Thus, the most "privileged" are those types that do not fork over the empty set \emptyset . If T is stable then, as we all know, non-forking types exist in sufficient abundance. If T happens also to have a group operation, then we are in the presence of a stable group.

The theory of stable groups has attracted much interest in recent years. There are several reasons for this interest. One is the fact that stable groups occur "in nature" more often than one might think. Another is that the theory of stable groups presents special features, due to the richer structure of the family of types. Indeed, the group itself acts on the family of types both from the left and from the right. If $p \in S(\mathbb{C})$ and g is an element of \mathbb{C} , then we define the left translate $gp \in S(\mathbb{C})$ of p by: $\varphi(x, \bar{a}) \in gp$ iff $\varphi(gx, \bar{a}) \in p$. In other words, gp is the type of any element of the form gc where c realizes the type p . The notion of right translate pg is defined analogously. Having these notions at hand, the following thought is quite natural: if $p \in S(\mathbb{C})$ does not fork over \emptyset , then it is a quite privileged type, but if it so happens that all its left translates also do not fork over \emptyset then p is truly privileged. More formally, p is called a left-generic type iff gp does not fork over \emptyset for all