COUNTABLY CATEGORICAL EXPANSIONS OF PROJECTIVE SPACES

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1. INTRODUCTION

A number of important problems in model theory ask what extra structure can be imposed upon a model M, while preserving various model-theoretic properties of M. For example, it has been conjectured that if extra structure is imposed upon an algebraically closed field F, then the resulting model F^+ no longer has finite Morley rank. In this paper, we shall discuss various open problems concerning ω-categorical structures of the form $M = \langle PG(\omega,q),R\rangle$. Here $PG(\omega,q)$ denotes an infinite dimensional projective space over the finite field GF(q) and R is some extra relation. Our starting point is the observation that structures of this form provide an interesting test case for Lachlan's conjecture that a stable ω-categorical structure is ω-stable.

Theorem 1.1

Suppose that $M = \langle PG(\omega,q),R\rangle$ is ω-stable and ω-categorical. If $G = \mathrm{Aut}\ M$ acts primitively on M, then M is strictly minimal.

Proof

By [8], M can be expressed as a union of finite algebraically closed subsets, $M = \bigcup_{i\in\omega} M_i$, such that

(i) $G_i = \mathrm{Aut}\ M_i$ acts primitively on M_i;

(ii) G_i has the same number n_2 of orbits on the lines of M_i as G has on the lines of M. Let $M_i = \langle P_i, R_i\rangle$, where P_i is a subspace of dimension d_i. (Throughout this paper, we will be using vector space dimension; so that

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