MODEL THEORY, STABILITY THEORY & STABLE GROUPS

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The aim of this chapter is to introduce the reader to the theory of stable groups not to give a rigorous exposition of the general theory. Thus we tend to proceed from the concrete to the abstract, with several examples and analyses of special cases along the way. On the other hand, getting to grips with stable groups presupposes some understanding of the point of view of model theory in general and stability theory in particular, and the first few sections are devoted to the latter.

1. MODEL THEORY

By a relational structure $\underline{M}$ we understand a set M (called the universe or underlying set of $\underline{M}$) equipped with relations R_i of arity $n_i < \omega$ say, for $i \in I$. Namely, for $i \in I$, R_i is a subset of the Cartesian product M^{n_i}. Here I and $<n_i : i \in I>$ depend on $\underline{M}$ and are called the signature of M. We also insist that I always contains a distinguished element $i_=$ such that $R_{i_=}$ is the diagonal $\{(a,a) : a \in M\} \subseteq M^2$. Often the distinction between M and $\underline{M}$ is blurred. The model theorist is interested in certain subsets of M and of M^n (the definable sets) which are obtained in a simple fashion from the R_i. So $\mathcal{D}(M)$ is a collection of subsets of M^n, $n< \omega$, which can be characterized as follows:

(i) Every $R_i \in \mathcal{D}(M)$.

(ii) If $n < \omega$, $X \in \mathcal{D}(M)$ is a subset of M^n and π is a permutation of $\{1,...,n\}$ then $\pi(X) = \{(a_{\pi(1)},...,a_{\pi(n)}) : (a_1,...,a_n) \in X\} \in \mathcal{D}(M)$.

(iii) $\mathcal{D}(M)$ is closed under Boolean combinations, i.e. if $n < \omega$ and

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