is uniformly continuous. Further he proves that the values of such a function on a closed interval are bounded and that the upper and lower bounds are attained. He also proves that such a function takes every value between two of its values. If a quasi-primary function has a derivative for every (primary) real number, then this derivative is again a quasi-primary function.

He also develops a theo ry of integration, defining first the Riemann integral, later also Lebesque's. It might seem that a measure theory must be impossible in this system, because by ordinary concepts the measure should be = 0 for denumerable sets, and here all sets are denumerable in a sufficiently high layer. However, the distinction between primary and secondary sets makes a definition of measure possible in such a way that the primary sets all get the measure 0, but not the secondary sets.

This system has one great advantage in distinction to the previous ones, namely, that the objects we are dealing with are all definitely and explicitly given. It is true of course that the unsolvability or even undecidability of many problems remains as before, but we know what we are talking about. In the previous theories it was at any rate not required that our considerations should be restricted to the definable or constructible objects.

16. Some remarks on intuitionist mathematics

Of great interest is the so-called intuitionism which above all is due to the Dutch mathematician L. E. J. Brouwer. This theory is essentially characterized by the requirement that an assertion of the existence of a mathematical object must contain a means of finding or constructing such an object. Further, the use of such a formal logical principle as "tertium non datur" is only justified, if we have a decision procedure. The intuitionist critique of classical mathematics is similar to the critique of Kronecker who also declared that a great part of ordinary mathematics was only words. It would lead too far, however, if I should give in these lectures a detailed exposition of the intuitionist foundation of mathematics. I must confine my exposition here to a few remarks which I hope will give an idea of the intuitionist way of reasoning.

The conjunction $p \,\&\, q$ retains its usual meaning also in intuitionist logic. The disjunction $p \vee q$ can be asserted if and only if either p can be asserted or q can. The negation $\neg p$ shall mean that the assumption p leads to a contradiction. The implication $p \rightarrow q$ means that we are in possession of a certain construction which will furnish a proof of q as soon as a proof of p is available. The assertion $(x)p(x)$ is justified if we possess a schema showing the property $p(x)$ for an arbitrary x, and $(E(x)p(x)$ can be asserted if we know an x with the property p or at least have a method for constructing such an x.

Since we have no general method to prove either p or $\neg p$, the tertium non datur, $p \vee \neg p$, is not generally valid. It can be proved that $p \rightarrow \neg\neg p$ is generally true, but not the inverse implication. Such differences in the propositional logic cause differences in predicate logic of course. As an interest-