If we turn to analysis it must be remarked that the classical form of it cannot be obtained. Indeed it will be necessary to distinguish between real numbers of different orders. A class of real numbers of 1. order which is bounded above possesses an upper bound, but this bound may then be a real number of order 2. Nevertheless a great part of analysis can be developed as usual, namely, the most useful part of it dealing with continuous functions, closed point-sets, etc. The reason for this is that it is often possible to prove theorems of reducibility, namely, theorems saying that a class (or relation) of a certain order coincides with one of lower order. I will not enter into this but only refer the reader to the book: "Das Kontinuum" by H. Weyl, where he has developed such a kind of predicative analysis.

15. Lorenzen's operative mathematics

In more recent years the German mathematician P. Lorenzen has set forth a system of mathematics which in some respects resembles the ramified theory of types, but it has also one important feature in common with the simple theory of types, namely, that the simple infinite sequence and similar notions are characterized by an induction principle which is assumed valid within all layers of objects. Lorenzen talks namely about layers of objects, not of types or orders. To begin with he takes into account some original objects, say numerals, figures built up in a so-called calculus as follows. We have the rules of production

$$1$$
$$k \rightarrow k1$$

which means that the object or symbol 1 is originally given and whenever we have a symbol or a string of symbols k we may build the string k 1 obtained by placing 1 after k. He introduces the notion "system". A system is a finite set of symbols. The systems are obtained by the rules

$$x$$
$$X \rightarrow X, x$$

The length or cardinal number of a system X is denoted by $|X|$. He gives the rules

$$|x| = 1$$
$$|X,x| = |X|\,1$$

for these lengths. Now the explanation of the successive layers of language is as follows.

From certain originally given symbols called atoms, say $u_1 \ldots\ldots u_n$, he constructs strings of symbols by the schema

$$x \rightarrow xu_1$$
$$\cdots\cdots$$
$$x \rightarrow xu_n$$