From this relation it follows (see the proof below) that

$$(2) \qquad \overline{\overline{EEV}} < \overline{\overline{EV}},$$

so that the sets V,EV, EEV, ... will possess decreasing cardinal numbers. The existence of such a decreasing sequence of cardinals shows that these cardinals cannot be alephs, whence it follows that not all sets can be well-ordered. Therefore, the axiom of choice cannot be added to the other axioms of Quine's theory without contradictions. We may express this fact by saying that the principle of choice can be proved false in Quine's theory. This was pointed out by Specker.

Proof that (2) follows from (1): Because of (1) there exists a mapping of the set of all unit sets $\{m\}$ on a subset of V. Indeed the identical mapping is of that kind. However, the identical mapping maps the set of all $\{\{m\}\}$ on just this subset of all sets $\{m\}$. Let us on the other hand assume that EV could be mapped onto EEV. The mapping would then consist of mutually disjoint pairs $(\{m\}, \{\{n\}\})$. However, the certainly existing set of pairs $(m, \{n\})$ would then furnish a mapping of V on EV contrary to (1). Hence (2) follows from (1).

The theory of Quine's does not seem to have many adherents among mathematicians. The reason for this is presumably the existence of such sets in it as V which are elements of themselves, pathological sets as they are called. I don't think, however, that this circumstance ought to worry mathematicians, because it is not necessary to take these abnormal sets into account in the development of the ordinary mathematical theories.

14. The ramified theory of types. Predicative set theory

I have already mentioned Poincare's objection to Cantor's set theory, that one makes use of the so-called non-predicative definitions. These definitions collect objects in such a way that the totality of these objects, or objects logically dependent upon that totality, are considered as belonging to the same totality, so that the definition has a circular character. It might perhaps be better to say that a non-predicative definition is the definition of an entity by a logical expression containing a bound variable such that the defined entity is one of the possible values of this variable. However, instead of trying to explain this generally, I think it is better to take a characteristic example.

Let us consider mankind, the domain of all human beings. We have the binary relation "x is a child of y" which I write Ch(x,y). Let us try to define descendant of P, P any given person. If we make use of the notion of finite number we may proceed thus: We define the relation $Ch^n(x,y)$ recursively by letting

$$Ch^1(x,y) \text{ stand for } Ch(x,y)$$

$$Ch^{n+1}(x,y) \text{ stand for } (Ez)(Ch^n(x,z)\ \&\ Ch(z,y)).$$