PREFACE TO THE SECOND EDITION

In the present second edition it has been decided not to make changes in the text; but the publishers have asked me to make here a brief statement of progress since the first edition appeared, and to comment on changes which would be desirable in the light of this progress.

When these lectures were delivered in April, 1948, there were several unanswered questions. Although work on these questions was continued while the manuscript was in press, yet an effort was made to make the text proper represent exactly the state of affairs at the time of delivery, with brief indications of later results in the footnotes. (The typing of the final copy for photography was completed in April, 1949, and minor corrections could be made until December, 1949.) The later results were then written up in [95], [96], [98], [106], and [108]; of these [95] and [108], ([98] is an abstract of these two) contain the new results in regard to negation; [96] contains a complete revision of Chapter V, including the elimination theorem for the systems LXY, as well as a reformulation and more abstract proof of the elimination theorem in general and its extension to the singular forms of LC and LK (described in V 7.3, p. 110); and [106] deals with the permutability of rules and the strengthened Gentzen Hauptsatz for the classical system. (This paper requires correction--see below.)

In addition to these papers, [103], which represents lectures delivered at Louvain in the winter of 1950-51, contains a treatment of that algebraic approach whose neglect was mentioned in the preface to the first edition, p. iv. Philosophical comment on the nature of implication, largely based on these lectures, is contained in [107] and [99]. Finally, in a proposed book [111] on which Prof. Feys and I are collaborating, applications of the Gentzen method to proving the consistency of certain systems of combinatory logic are expected to appear. The fact that such methods could be used was already mentioned in [16] and [17], and formed part of the motivation for making this study in the first place. In particular [111], in §9F4 contains the proof of a form of elimination theorem; if this method of proof were adapted to the present circumstances it would give a proof of the elimination theorem which would be valid, under certain limitations, without the rules W and K; this would be a great improvement on the proof given here or in [96]. However, this has not yet been worked out in detail.

In the meantime Gentzen's methods, and others similar to them, have interested several other writers. A French translation of Gentzen's thesis by Feys and Ladrière [112] has recently