

ON THE RELATION BETWEEN  
CALCULUS OF PROBABILITY AND STATISTICS

by

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In a paragraph following his elegant presentation of the Calculus of Probability, Professor Copeland touches on the problem of the application of the theory to actual observations. In the sequel we shall elaborate further on the connection between the calculus of probability and statistics in order to establish a bridge from Copeland's exposition to Wald's introduction to modern statistics contained in Number 1 of the Notre Dame Mathematical Lectures.

In applying the concept of  $p(x)$  to sequences of observations, we encounter the difficulty that all sequences of observations are finite whereas  $p(x)$  refers to the infinite sequence  $x$  of Zeros and Ones. By definition,  $p(x)$  is the limit of the relative frequencies of Zeros in the finite initial segments of the sequence  $x$ , as these segments increase in length. Clearly, the knowledge of a finite initial segment of  $x$ , no matter how long the segment may be, does not permit any apodictical conclusion concerning the relative frequency in any more extended initial segment or concerning the limit  $p(x)$  of these relative frequencies. In other words, logically, each finite initial segment of  $x$  is compatible with each hypothesis concerning  $p(x)$ . Or, in still other words, on the basis of a finite sequence of observations we can neither assert nor