IV THE THEORY OF CONFIDENCE INTERVALS

The procedure of estimation, as I formulated it here, is also called estimation by a point. For practical applications the estimation by intervals seems to be much more important. That is to say, we have to construct two functions of the observations $\underline{\theta}(E)$ and $\overline{\theta}(E)$, where E denotes a point of the sample space, and we estimate the parameter to be within the interval $\delta(E) = \left[\underline{\theta}(E), \overline{\theta}(E)\right]$. In connection with the theory of interval estimation, R. A. Fisher introduced the notion of fiducial probability and fiducial limits, while Neyman[8] developed the theory of interval estimation based on the classical theory of probability. I shall give here a brief outline of Neyman's theory.

Before the sample has been drawn the point E is a random variable and, therefore, the values of $\underline{\theta}(E)$ and $\overline{\theta}(E)$ are also random variables. Hence, before the sample has been drawn we can speak of the probability that

$$(3) \qquad \underline{\theta}(E) \leq \theta \leq \overline{\theta}(E)$$

even if θ is considered merely as an unknown constant. After the sample has been drawn and we have obtained a particular sample point, say E_0, it does not make sense to speak of the probability that

$$(4) \qquad \underline{\theta}(E_0) \leq \theta \leq \overline{\theta}(E_0),$$

if θ is merely an unknown constant. Each term in the inequality (4) is a fixed constant, and the inequality (4) is either

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8) See reference 15