III R. A. FISHER'S THEORY OF ESTIMATION[5)]

The problem of estimation of the unknown parameter θ is the problem of finding a function $t(x_1,\ldots,x_n)$ of the observations such that t can be considered in a certain sense as a "good" or "best" estimate of θ. Since the estimate $t(x_1,\ldots,x_n)$ is a random variable, we cannot expect that its value should coincide with that of the unknown parameter, but we will try to choose $t(x_1,\ldots,x_n)$ in such a way as to make as great as possible the probability of the value of t lying as near as possible to the value of the unknown parameter θ.

This is a somewhat vague formulation of the requirement for a "good" or "best" statistical estimate. It can be made precise in different ways. Markoff[6)], for instance, defines the notion of a "best" estimate as follows: A statistic t (we shall call any function of the observations a statistic) is a best estimate of θ if

(1) t is an unbiased estimate of θ, i.e., $E_\theta(t) \doteq \theta$ identically in θ where $E_\theta(t)$ denotes the expected value of t under the assumption that θ is the true value of the parameter.

(2) $E_\theta(t-\theta)^2 \leq E_\theta(t'-\theta)^2$ identically in θ for all t' which satisfy (1).

This definition of a "best estimate" seems to be a reasonable and acceptable one since, in general, the smaller the variance of t the greater is the probability that t will lie in a small

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5) See references 3 - 6

6) See reference 15, p.344