

# Contents

<b>I PRELIMINARY CONCEPTS</b>	<b>1</b>
<b>1 Frechet manifolds</b>	<b>3</b>
1.1 Frechet vector spaces . . . . .	3
1.1.1 Basic definitions . . . . .	3
1.1.2 Derivative . . . . .	5
1.2 Frechet manifolds . . . . .	7
1.2.1 Basic definitions . . . . .	7
1.2.2 Frechet vector bundles . . . . .	9
1.2.3 Connections . . . . .	11
1.2.4 Differential forms . . . . .	14
1.2.5 Symplectic and complex structures . . . . .	16
<b>2 Frechet Lie groups</b>	<b>19</b>
2.1 Group of currents . . . . .	20
2.1.1 Basic properties . . . . .	20
2.1.2 Exponential map of the loop algebra . . . . .	22
2.1.3 Complexification . . . . .	23
2.2 Group of diffeomorphisms . . . . .	24
2.2.1 Finite-dimensional subalgebras in $\text{Vect}_\omega(S^1)$ . . . . .	25
2.2.2 Exponential map of $\text{Vect}(S^1)$ . . . . .	27
2.2.3 Simplicity of $\text{Diff}_+(S^1)$ . . . . .	30
<b>3 Flag manifolds and representations</b>	<b>35</b>
3.1 Flag manifolds . . . . .	35
3.1.1 Geometric definition of flag manifolds . . . . .	35
3.1.2 Borel and parabolic subalgebras . . . . .	37
3.1.3 Algebraic definition of flag manifolds . . . . .	39
3.2 Irreducible representations . . . . .	41
3.2.1 Irreducible representations of complex semisimple Lie groups . . . . .	41
3.2.2 Borel–Weil construction . . . . .	43
3.2.3 Orbit method and coadjoint representation . . . . .	44
<b>4 Central extensions and cohomologies</b>	<b>49</b>
4.1 Central extensions of Lie groups and projective representations . . . . .	49
4.2 Cohomologies of Lie algebras . . . . .	51
4.3 Cohomologies of Lie groups . . . . .	54