Preface to Volume 1.

This is the first volume of a projected series of monographs to appear under the unified title "On Congruence Monodromy Problems." The main content of this volume (and a part of the next one) is based on my lectures given at Princeton University in the spring term of 1967, and at the University of Tokyo in the fall and the winter term, 1967-68.¹ A brief summary of the main content is given without proofs in [18], which will appear in the Journal of Mathematical Society of Japan 20-1, 1968.

My aim in this series is to solve the problems presented in the title, which are described briefly in the introduction and in more detail in [18]. To this end, several different approaches are required, and they are introduced separately in different chapters. Therefore, in this volume where each approach is still in its beginning stage, each chapter is almost independent of another. On the other hand, each chapter will be continued on to the successive volumes, so that we may present any further development of the approach introduced in that chapter.

Brief explanations on the relations between our problems and results are given in [18]. Here, I wish to express my sincere thanks to the professors of the above mentioned universities for giving me the occasions to lecture on this nascent subject, and to the professors and colleagues of the universities who attended my lectures with patience and gave me several useful remarks. I would also like to acknowledge some valuable remarks given by Professors R. P. Langlands and H. Shimizu (some of which corrected my errors), and encouragement and stimulation provided by Professors Y. Kawada and A. Weil. Now, my deepest acknowledgment is to Professor G. Shimura for his interest in this subject, constant encouragement and most valuable discussions. I regret that in this volume I could not make clear the significance of his results [32] on Shimura-curves² and their relations to our problems. Actually, there are almost no relations at this stage of our problems and results; however, at the following stages of Chapter 4, his results [32] will play an essential role. At any rate, I have received much encouragement and stimulation (and entertained some further conjectures) from his results. Finally, I am sincerely thankful to Miss C. Sagawa in the University of Tokyo for typing this lengthy monograph.

Tokyo, Japan, February 1968, Y. IHARA

¹This is mainly based on the former. However, Chap. 2 §§35, 36; Chap. 3 §10- §14; and Chap. 4 §§4, 5 are newly added, and Chap. 2 was moreover considerably revised. In Tokyo, Chap. 1 and Chap. 5 (including the forthcoming Part 2) were given with more expositions.

²I.e., algebraic curves uniformized by automorphic functions attached to quaternion algebras over totally real algebraic number fields, whose beautiful arithmetic properties have been discovered by Shimura (in [32]); before him, nothing was known about them except in the elliptic modular case.