Notations and symbols

Here are notations and symbols frequently used in this monograph.

- $\mathbb{N}$ = the set of all natural numbers. In this monograph, natural number means positive integer. Thus $0 \notin \mathbb{N}$.

  $\mathbb{Z}$ = the set of all integers.

  $\mathbb{Q}$ = the set of all rational numbers.

  $\mathbb{R}$ = the set of all real numbers.

  $\mathbb{C}$ = the set of all complex numbers.
- $\mathcal{B}(\mathbb{R}^n)$ = the Borel σ-algebra of $\mathbb{R}^n$, $\mathcal{B}(\mathbb{C})$ = the Borel σ-algebra of $\mathbb{C}$.
- We denote the *imaginary unit* by $\sqrt{-1}$. The letter 'i' is not used for the imaginary unit, because we wish to use this as an index. For $z \in \mathbb{C}$, let

$$\begin{aligned} \operatorname{Re} z &= \text{the } \textit{real part} \text{ of } z, \\ \operatorname{Im} z &= \text{the } \textit{imaginary part} \text{ of } z, \\ \overline{z} &= \text{the } \textit{conjugate} \text{ of } z. \end{aligned}$$

- μ = the 1-*dimensional Lebesgue measure*.
- For $a, b \in \mathbb{R}$, let

$$\begin{aligned} &a \vee b = \max\{a, b\}, \ \ a \wedge b = \min\{a, b\}, \\ &a^+ = a \vee 0, \ \ a^- = (-a)^+ = (-a) \vee 0. \end{aligned}$$

- For $A \subset X$ where X is a universal set,

$$\mathbf{1}_A(x) = \begin{cases} 1, & x \in A, \\ 0, & x \in X \setminus A, \end{cases}$$

$$\begin{aligned} A^{\complement} &= X \setminus A \\ &= \{x \in X; x \notin A\}. \end{aligned}$$

  $\mathbf{1}_A$ is called the *defining function* (or *indicator function*) of A, and $A^{\complement}$ the *complement* of A.
- For a set A, card A = the *cardinality* of A. Let $\aleph_0$ = card $\mathbb{N}$ and $\aleph$ = card $\mathbb{R}$. $\aleph_0$ is called the *countable infinite cardinality* and $\aleph$ the *cardinality of the continuum*. When card $A \leq \aleph_0$, i.e., A is at most countable, this is written as $\#A$. $\#A$ is the number of elements of A.