Quantum Many-Body Problems and Representation Theory