

Appendix A Source code of Ordering Algorithm

For the reader's convenience we present a source code of Ordering Algorithm (Algorithm 5.7) written for SageMath 9.4¹. It is an almost direct translation of Algorithm 5.7, and no effort is made for the efficiency and sophistication. (If any minor or major bug is found, please let us know.)

The following examples explain how to use it on Sage Notebook.

Example A.1. (a). Let us order the product $\Psi[e_2]^5\Psi[e_1]^3$ modulo $G^{>5}$.

In:

```
order(5, [[0,1,5], [1,0,3]])
```

Out:

```
[[1, 0, 3],
 [3, 1, 5],
 [2, 1, 15],
 [3, 2, 125],
 [1, 1, 15],
 [2, 2, 60],
 [2, 3, 270],
 [1, 2, 30],
 [1, 3, 30],
 [1, 4, 15],
 [0, 1, 5]]
```

(b). One can order any (not necessarily anti-ordered) product with factors of the form $\Psi[tn]^{s/t}$ ($n \in N_{\text{pr}}^+$, $s, t \in \mathbb{Z}_{>0}$) as specified in Algorithm 5.7.

In:

```
order(6, [[0,1,1], [1,2,1], [2,1,1], [2,2,1/2], [1,0,2]])
```

Out:

```
[[1, 0, 2],
 [4, 1, 1],
 [3, 1, 2],
 [2, 1, 2],
 [4, 2, 19],
 [3, 2, 16],
 [1, 1, 2],
 [2, 2, 13/2],
 [3, 3, 33],
 [2, 3, 10],
 [1, 2, 1],
 [2, 4, 9/2],
 [1, 3, 1],
 [0, 1, 1]]
```

Enjoy exploring the Badlands!

¹Sage Mathematics Software, The Sage Development Team, <https://www.sagemath.org>.