

Preface

This monograph is based on notes for my lectures given at Osaka University in June, 2011, at Kumamoto University in October, 2013, and at Tsukuba University in January, 2015. The subject of this monograph is the Cauchy problem for (systems of) nonlinear wave equations, mostly with small initial data.

After the pioneering works of John in 1970s, nonlinear wave equations with small initial data were widely studied. In 1980s, Klainerman introduced the so-called vector field method (or the invariant norm method), and obtained the global existence results for small initial data, as well as the lifespan estimates. He also introduced the null condition to ensure the global existence of classical solutions for systems of nonlinear wave equations with quadratic nonlinearity in three space dimensions. The vector field method was improved and extended by many authors. In 2000s, Lindblad and Rodnianski introduced a notion of the weak null condition, and Alinhac obtained some global existence results under weaker conditions than the null condition.

In this monograph, the Cauchy problem for systems of nonlinear wave equations with small initial data is considered. After showing the classical local existence theorem, we discuss sufficient conditions for the small data global existence, the asymptotic behavior of global solutions, and the lifespan estimates of solutions, with an emphasis on the relation between the nonlinear wave equations and their reduced equations. Some recent progress on the global existence and the asymptotic behavior of solutions under a weaker condition than the null condition is also included.

The readers are assumed to have some familiarity with certain basic facts on the Lebesgue integrals, the distribution theory including Fourier analysis, and functional analysis. Beyond these basic matters, this monograph is intended to be self-contained, at least for the main stories. Especially, no knowledge on linear and nonlinear wave equations are assumed.

Choice of the topics depends on my taste, and many important topics, such as local existence for rough initial data, bilinear estimates, Strichartz estimates, global solutions for large initial data, and the mixed problems, are missing. There are so many important works in this field, and the references are not intended to be complete.

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