Guide for the reader

In Chapter 1, we start with Mori's cone theorem for smooth projective varieties and his contraction theorem for smooth threefolds. It is one of the starting points of the minimal model program. So the minimal model program is sometimes called Mori's program. We also explain some examples of quasi-log schemes, the motivation of our vanishing theorems, the background of this book, the author's related papers, and so on, for the reader's convenience. Chapter 2 collects several definitions and preliminary results. Almost all the topics in this chapter are well known to the experts and are indispensable for the study of the minimal model program. We recommend the reader to be familiar with them. We note that the invariant Iitaka dimension introduced by Sung Rak Choi has not been treated in the standard literature. To the best of our knowledge, we can not find the definition and the basic properties of relatively big $\mathbb{R}$-divisors on non-normal irreducible varieties in the literature. Therefore, Chapter 2 may be useful to the experts. In Chapter 3, we discuss various Kodaira type vanishing theorems and several applications. Although this chapter contains several new results and arguments, almost all the results are standard and are known to the experts. Chapter 4 is a survey on the minimal model program. We discuss the basic results of the minimal model program, the recent results by Birkar–Cascini–Hacon–M$^{\mathrm{c}}$Kernan, and various results on log canonical pairs, log surfaces, semi-log canonical pairs by the author, and so on, without proof. Chapter 5 is devoted to some injectivity, vanishing, and torsion-free theorems for *reducible* varieties. They are generalizations of Kollár's corresponding results from the mixed Hodge theoretic viewpoint and play crucial roles in the theory of quasi-log schemes. Chapter 6 is the main part of this book. We prove the adjunction and the vanishing theorem for quasi-log schemes as applications of the results in Chapter 5. Then we establish the basepoint-free theorem, the rationality theorem, and the cone theorem for quasi-log schemes, and so on. Chapter 7 collects some supplementary results and examples. We recommend the reader who is familiar with the traditional minimal model program and is only interested in the theory of quasi-log schemes to go directly to Chapter 6.