

Contents

1	Introduction	1
1.1	Mori’s cone and contraction theorem	1
1.2	What is a quasi-log scheme?	3
1.3	Motivation	5
1.4	Observation	8
1.5	Background	11
1.6	Comparison with the unpublished manuscript	13
1.7	Related papers	13
1.8	Notation and convention	14
2	Preliminaries	16
2.1	Divisors, \mathbb{Q} -divisors, and \mathbb{R} -divisors	16
2.2	Kleiman–Mori cones	26
2.3	Singularities of pairs	29
2.4	Iitaka dimensions, movable and pseudo-effective divisors	39
2.5	Iitaka dimensions for \mathbb{R} -divisors	42
3	Classical vanishing theorems and some applications	48
3.1	Kodaira vanishing theorem	49
3.2	Kawamata–Viehweg vanishing theorem	54
3.3	Viehweg vanishing theorem	61
3.4	Nadel vanishing theorem	66
3.5	Miyaoka vanishing theorem	67
3.6	Kollar injectivity theorem	69
3.7	Enoki injectivity theorem	70
3.8	Fujita vanishing theorem	74
3.9	Applications of Fujita vanishing theorem	81
3.10	Tanaka vanishing theorems	84
3.11	Ambro vanishing theorem	85
3.12	Kovács’s characterization of rational singularities	87
3.13	Basic properties of dlt pairs	89
3.14	Elkik–Fujita vanishing theorem	96
3.15	Method of two spectral sequences	100
3.16	Toward new vanishing theorems	102
4	Minimal model program	106
4.1	Fundamental theorems for klt pairs	107
4.2	X-method	108