Structure of this monograph

Since Maruyama's manuscript was left without Introduction, the beginning paragraphs of each chapter are excerpted here for readers' convenience.

Chapter I: First we shall show that if we collect all the vector bundles on a projective variety and require a weak universal property of a moduli space, then there does not exist the moduli space. This motivates us to introduce the notion of stability and semi-stability. The idea of Harder-Narasimhan filtration plays a crucial role sometimes behind strong results and sometimes very explicitly. Two of basic results on boundedness are proved in the section 3. The formulation of the first is due to L. S. Kleiman [K2] and the second is a theorem of Grothendieck [G]. We shall show a beautiful application of the second result in the proof of the openness of stability.

Chapter II. Is the semi-stability preserved when we make the restriction of a semi-stable sheaf to a hyperplane section? We have to, of course, put some conditions on semi-stable sheaves and hyperplanes. What kind of conditions then? We shall show here several types of restriction theorems and apply them to proving results on boundedness of semi-stable sheaves.

Chapter III: We are now going to construct the moduli spaces of stable sheaves. If we have a bounded family of stable sheaves, it can be parameterized by an open subscheme of a Quot-scheme on which a reductive group scheme acts. Two points of the subscheme correspond to the same sheaf if and only if they are in the same orbit of the group scheme action. Thus main part of the construction is to prove the existence of a quotient scheme of the scheme by the group scheme. Once we can overcome this difficulty, the universality of the moduli space is easily derived from that of the Quot-scheme and the quotient.

In the first section we shall recall a proof of the representability of the Quot-functors. We shall recall the results in Geometric Invariant Theory that are relevant to our aim. The most of results are only stated without proof and the author refers the readers to the famous "Geometric Invariant Theory" by D. Mumford and Seshadri's work [Se]. After the study of rather technical notion of e-stability, the proof of a fundamental lemma to connect semi-stable sheaves with semi-stable points in Geometric Invariant Theory and the preparation of the general setting, we are going to construct moduli space of stable-sheaves. To compactify the moduli spaces we have to add points corresponding to the S-equivalence classes of semi-stable sheaves. The case over a field of characteristic zero is treated in the sixth section along a beautiful idea of Simpson. To handle semi-stable sheaves in the general cases, we need a detailed analysis of closed orbits of a Grassmann type scheme. The seventh section is devoted to this analysis. A proof of the existence of moduli spaces in general cases will be given in the last section.

To construct the moduli spaces of semi-stable sheaves on a singular variety, Simpson's idea in [S] seems to be inevitable and it is essential throughout this chapter.