Foreword

Masaki Maruyama, professor emeritus of Kyoto University and professor of Doshisha University, died of cancer in Kyoto on April 10, 2009, at the age of 64 years. This is a posthumous publication of his manuscript left in a personal computer in his study.

Maruyama began his study of (algebraic) vector bundles on algebraic curves with the joint article [1] with Masayoshi Nagata and his master thesis [2]. In 1972, he was awarded Doctor of Science from Kyoto University by his doctoral dissertation [3]. Since then he has generalized the theory to those on higher dimensional algebraic varieties. Especially he made pioneering works on the moduli space of vector bundles on algebraic varieties, or more generally, coherent sheaves on schemes. The first work of his in this direction is [4] and its main part was done during Maruyama stayed in Harvard University in the academic year 1972–73. The moduli spaces, constructed in [5], are sometimes called the Gieseker-Maruyama space, expressing respect to him. The posthumous manuscript, which seems to be prepared in 90's, summarizes his lifework in this field. My personal view of this field and on some features of this monograph is supplied in the following since his manuscript had no preface or introduction.

In algebraic geometry, the birational invariants of algebraic varieties are crucially important in classifying them. The irregularity, usually denoted by q, is such an invariant and classically defined as the dimension of the Picard variety, which is an indispensable moduli space equivalent to the class group of a number field. In fact, the Picard variety is the moduli space of invertible sheaves, or vector bundles of rank one, on an algebraic varieties. It is an algebraic variety which parametrizes the isomorphism classes of invertible sheaves *most economically.* (See Chapter I for precise explanation of moduli.)

In view of importance of the Picard variety, many mathematicians had tried to generalize the Picard variety to the moduli space of vector bundles of higher rank but there were both technical and essential difficulties. They could not give a reasonably natural algebraic structure on the set of all isomorphism classes of vector bundles of higher rank[1]. This was a puzzle for long years and two discoveries were required to solve it, besides the progress of algebraic geometry itself. One is the notion of stability, first introduced by David Mumford in his famous book which is called GIT for short and *Bible* in this text of Maruyama's. In the invited talk of ICM in Stockholm 1962, Mumford proposed the following definition.

($*$) A rank two vector bundle E on a smooth complete algebraic curve is *stable* if the inequality $\deg L < \frac{1}{2} \deg E$ holds for every line subbundle $L \subset E$.

It is important that this simple-looking definition was discovered via computa-

[1]There's also another very different approach which loosen the notion of *algebraic structure* from scheme to another one, say, (Artin's) algebraic space. Nowadays algebraic stack, a much weaker notion, is used as an algebraic structure put on a certain category of vector bundles in place of the set of isomorphism classes.