Preface

This is a volume of lecture notes based on three series of lectures given by three visiting professors of RIMS, Kyoto University during the year long project research "Discrete Geometric Analysis" in the Japanese academic year 2012. The aim of the project research was to make comprehensive research on topics related to discreteness in geometry, analysis and optimization. The goal is to expand and make a new stream of discrete geometric analysis by exchanging ideas in each area. The main themes during the project and also of this volume are threefold: i) Discrete probability theory and analysis on graphs, ii) Discrete convex analysis and its applications, iii) Geometric group theory. We had five conferences at Kyoto University in the project research. The three visiting professors gave the series of lectures on the following topics.

(I) Loop Erased Walks and Uniform Spanning Trees,
by Martin T. Barlow (University of British Columbia).

(II) Combinatorial Rigidity: Graphs and Matroids in the Theory of Rigid Frameworks,
by Tibor Jordán (Eötvös University).

(III) Analysis and Geometry on Groups,
by by Andrzej Zuk (Université Paris 7).

Discrete geometric analysis is a hybrid field of several traditional disciplines; say, graph theory, geometry, theory of discrete groups, and probability. The terminology "Discrete Geometric Analysis" was invented by Toshikazu Sunada. (See the survey by Sunada [TS] for his original ideas of this field. See also [DGA, PAG] for related proceedings.) Discrete geometric analysis has been extending and making new interactions in many fields.

Let us discuss some details on each topic.

(I) Loop erased walks and uniform spanning trees: One of the main streams of the contemporary probability theory is to analyze discrete models and to discuss their scaling limits. The uniform spanning tree (UST) has been a very important fundamental model over the last few decades partly because the scaling limit of UST led Oded Schramm to introduce the SLE (stochastic Loewner evolution, now referred as the Schramm Loewner evolution) in his celebrated paper in 2000. A UST on a finite graph is a random graph that chooses a spanning tree (i.e. a graph without a cycle that uses all the vertices of the original graph) uniformly at random among all the choices of the spanning trees on the graph, and the UST on an infinite graph is defined as its local limit. The loop erased random walk (LERW) is a process that is produced by erasing loops of the random walk chronologically, that was introduced by Lawler in his 1980 paper. The two models relate in an intrin-