Introduction

Secondary characteristic classes are one of the most invariant tools in studying foliations. The Godbillon–Vey class is the most significant, and is extensively studied. It is well-known that it admits continuous deformations [65], [41]. On the other hand, when foliations are assumed to admit certain transversal structures, it often become rigid or trivial [12], [63], [20], [14], [44], [57]. In this monograph, secondary characteristic classes (secondary classes for short) of transversely holomorphic foliations are studied. It is known that the Godbillon–Vey class is decomposed into a product of another secondary class and a power of the Chern class ([64], [3]). We will call this secondary class the imaginary part of the Bott class. The Bott class is a complex secondary class in the sense that it is an invariant of transversely holomorphic foliations, and that it is a cohomology class with coefficients in $\mathbb{C}$ or $\mathbb{C}/\mathbb{Z}$. The definition of the Bott class is quite similar to that of the Godbillon–Vey class, however, it can be found already in [17] (see also [18, p. 49]). A lot of examples of non-trivial Godbillon–Vey classes are known for real foliations [65], [41]. On the other hand, such examples are barely known for transversely holomorphic foliations. There is a paper of Rasmussen [64], where some examples are given by using actions of complex Lie groups such as $\mathrm{SL}(n;\mathbb{C})$. The construction makes use of the existence of certain lattices of which the existence seems unknown [51]. In this monograph, we will first show that this difficulty can be avoided in a natural way, and construct some examples of the same kind by using other semisimple Lie groups. More precisely, we will show the following in Chapter 3.