Preface

1) Let $\big(\Omega = \mathcal{C}(\mathbb{R}_+ \to \mathbb{R}), (X_t, \mathcal{F}_t)_{t\geq 0}, \mathcal{F}_\infty = \bigvee_{t\geq 0} \mathcal{F}_t, W_x(x \in \mathbb{R})\big)$ denote the canonical realisation of one-dimensional Brownian motion. With the help of Feynman-Kac type penalisation results for Wiener measure, we have, in [RY, M], constructed on $(\Omega, \mathcal{F}_\infty)$ a positive and σ-finite measure $\mathbf{W}$. The aim of this second monograph, in particular Chapter 1, is to deepen our understanding of $\mathbf{W}$, as we discuss there other remarkable properties of this measure.
For pedagogical reasons, we have chosen to take up here again the construction of $\mathbf{W}$ found in [RY, M], so that the present monograph may be read, essentially, independently from our previous papers, including [RY, M].
Among the main properties of $\mathbf{W}$ presented here, let us cite :
• the close links between $\mathbf{W}$ and probabilities obtained by penalising Wiener measure by certain functionals : see Theorems 1.1.2, 1.1.11 and 1.1.11' ;
• the existence of integral representation formulae for the measure $\mathbf{W}$: see Theorems 1.1.6 and 1.1.8. These formulae allow to express $\mathbf{W}$ in terms of the laws of Brownian bridges and of the law of the 3-dimensional Bessel process (see formula (1.1.43)). They also allow to express $\mathbf{W}$ in terms of the law of Brownian motion stopped at the first time when its local time at 0 reaches level l, l varying, and of the law of the 3-dimensional Bessel process (see formula (1.1.40)). One may observe that these representation formulae are close to those obtained by Biane and Yor in [BY] for some different σ-finite measures on Wiener path space.
• the existence, for every $F \in L^1_+(\mathcal{F}_\infty, \mathbf{W})$, of a $\big((\mathcal{F}_t,\ t \geq 0), W\big)$ martingale $\big(M_t(F),\ t \geq 0\big)$ which converges to 0, as $t \to \infty$ (see Theorem 1.2.1). Many examples of such martingales are given (see Chap. 1, Examples 1 to 7). The Brownian martingales of the form $\big(M_t(F),\ t \geq 0\big)$ are characterized among the set of all Brownian martingales (see Corollary 1.2.6) and a decomposition theorem of every positive Brownian supermartingale involving the martingales $\big(M_t(F),\ t \geq 0\big)$ is established in Theorem 1.2.5. In the same spirit, we show (see Theorem 1.2.11) that every martingale $\big(M_t(F),\ t \geq 0\big)$ with $F \in L^1(\mathcal{F}_\infty, \mathbf{W})$, F not necessarily ≥ 0, may be decomposed in a canonical manner into the sum of two quasi-martingales which enjoy some remarkable properties. In particular, this result allows to obtain a characterization of the martingales $\big(M_t(F),\ t \geq 0\big)$, with $F \in L^1(\mathcal{F}_\infty, \mathbf{W})$ which vanish on the zero set of the process $(X_t,\ t \geq 0)$. This is Theorem 1.2.12.

• a general penalisation Theorem, for Wiener measure, which is valid for a large class $\mathcal{C}$ of penalisation functionals $(F_t,\ t \geq 0)$ and whose proof hinges essentially upon some remarkable properties of $\mathbf{W}$: this is the content of Subsection 1.2.5 and particularly Theorem 1.2.14 and Theorem 1.2.15.
• the existence of invariant measures, which are intimately related with $\mathbf{W}$, for several Markov processes taking values in function spaces (see Section 1.3). Chapter 1 of this monograph is devoted to the results we have just described.

2) The results relative to the 1-dimensional Brownian motion are extended, in Chapter 2 of this monograph to 2-dimensional Brownian motion (we identify $\mathbb{R}^2$ to $\mathbb{C}$, and use complex notation). In this framework, the role of the measure $\mathbf{W}$ is played by a positive and σ-finite measure, which we denote $\mathbf{W}^{(2)}$ on $\big(\Omega = \mathcal{C}(\mathbb{R}_+ \to \mathbb{C}),\ \mathcal{F}_\infty\big)$. The properties of $\mathbf{W}^{(2)}$ are, mutatis mutandis, analogous to those of $\mathbf{W}$. However, in the set-up of the $\mathbb{C}$-valued Brownian