General Introduction.

Here, only a general introduction is given. The content of each chapter is summarized in the introduction to each chapter.

THE PROBLEMS .[5] Let $G = PSL_2(\mathbf{R}) \times PSL_2(k_\mathfrak{p})$, where $\mathbf{R}$ and $k_\mathfrak{p}$ are the real number field and a $\mathfrak{p}$-adic number field with $N\mathfrak{p} = q$ respectively, and $PSL_2 = SL_2/\pm 1$. Let Γ be a torsion-free discrete subgroup of G with compact quotient, having a dense image of projection in each component of G. Our subject is such a discrete subgroup Γ. This study was motivated by the following series of conjectures which were suggested by our previous work **[15]**. Since our group Γ is essentially non-abelian (see Chapter 3, Theorem 2 in §6), the readers will see that, by our conjectures, Γ would describe a "*nonabelian class field theory*" over an algebraic function field of one variable with finite constant field $\mathbf{F}_{q^2}$. We would like to call the problems of determining the validity of these conjectures, *the congruence monodromy problems*.

CONJECTURES . With each Γ, we can associate an algebraic function field K of one variable with finite constant field $\mathbf{F}_{q^2}$ and with genus $g \geq 2$, and a finite set $\mathfrak{S}(K)$ consisting of $(q-1)(g-1)$ prime divisors of K of degree one over $\mathbf{F}_{q^2}$ satisfying the following properties. Here the elements of $\mathfrak{S}(K)$ are called the *exceptional prime divisors*, while all other prime divisors of K are called the *ordinary prime divisors.*

CONJECTURE 1. The ordinary prime divisors P of K are in one-to-one correspondence with the pairs $\{\gamma_P\}_\Gamma^\pm$ of mutually inverse primitive elliptic conjugacy classes of Γ (See Chapter 1, §§1-12 for the definitions).

CONJECTURE 2. The finite unramified extensions K' over K, in which all $(q-1)(g-1)$ exceptional prime divisors of K are decomposed completely, are in one-to-one correspondence with the subgroups Γ' of Γ with finite indices. Moreover, this one-to-one correspondence satisfies the Galois theory.

CONJECTURE 3. The law of decomposition of ordinary prime divisors P of K in K' is described by the corresponding $\{\gamma_P\}_\Gamma^\pm$ and Γ'. Namely, decompose the Γ-conjugacy class $\{\gamma_P\}_\Gamma$ into a disjoint union of Γ'-conjugacy classes:

$$\{\gamma_P\}_\Gamma = \{\gamma_{P,1}\}_{\Gamma'} \cup \cdots \cup \{\gamma_{P,t}\}_{\Gamma'},$$

and for each i, let f_i be the smallest positive integer such that $\gamma_{P,i}^{f_i}$ is contained in Γ'. Then we have $\sum_{i=1}^t f_i = (\Gamma : \Gamma')$, and our conjecture asserts that the decomposition of P in K' is

[5] Here, we shall reproduce a part of the introduction of my paper **[18]**. As for the details, cf. **[18]** §3.