Preface

The historical roots of the theory of bounded cohomology stretch back at least as far as Poincaré [**167**] who introduced rotation numbers in his study of circle diffeomorphisms. The Milnor–Wood inequality [**154, 204**] as generalized by Sullivan [**193**], and the theorem of Hirsch–Thurston [**109**] on foliated bundles with amenable holonomy groups were also landmark developments.

But it was not until the appearance of Gromov's seminal paper [**97**] that a number of previously distinct and isolated phenomena crystallized into a coherent subject. In [**97**] and in [**98**] Gromov indicated how many important or delicate geometric and algebraic properties of groups could be encoded and (in principle) recovered from their bounded cohomology. The essence of bounded cohomology is that it is a functor from the category of groups and homomorphisms to the category of normed vector spaces and norm-decreasing linear maps. Theorems in bounded cohomology can be restated as algebraic or topological inequalities; rigidity phenomena arise when equality is achieved (see e.g. [**31, 93, 149, 45**]).

A certain amount of activity followed; for example, the papers [**6, 27, 115, 150**] contain significant new ideas and advanced the subject. But there is a sense in which the promise of the field as suggested by Gromov has not been realized. One major shortcoming is the lack of adequate tools for computing or extracting meaningful information. There are at least two serious technical problems:

(1) The failure of the standard machinery of homological algebra (e.g. spectral sequences) to carry over to the bounded cohomology context in a straightforward way
(2) The fact that in the cases of most interest (e.g. hyperbolic groups) bounded cohomology is usually so big as to be unmanageable

Monod's monograph [**157**] addresses in a very useful way some of the most serious shortcomings of the subject by largely restricting attention to *continuous* bounded cohomology in contexts where this restriction is most informative. Burger and Monod (see especially [**33**] and [**34**]) developed the theory of continuous bounded cohomology into a powerful tool, which is of most value to people working in ergodic theory or the theory of lattices (especially in higher-rank) but is less useful for people whose main concern is the bounded cohomology of discrete groups (although Theorem 2 from [**34**] is an exception).

To get an idea of the state of the subject *ca.* 2000, we quote an excerpt from Burger–Monod [**35**], p. 19:

> Although the theory of bounded cohomology has recently found many applications in various fields ... for discrete groups it remains scarcely accessible to computation. As a matter of fact,