

Table of Contents

1	Introduction	1
2	Preliminaries from functional analysis	4
2.1	Overview	4
2.2	Linear operators in Banach spaces	4
2.3	Symmetric strictly monotone operators on Hilbert space	10
2.4	Basic interpolation theorems	16
3	Fourier transform and Sobolev spaces on flat space	20
3.1	Overview	20
3.2	Preliminary facts about holomorphic functions	20
3.3	Distributions and Fourier transform in Euclidean space	26
3.4	Homogeneous distributions and Fourier transform of concrete functions	33
3.5	Sobolev spaces of integer order	36
3.6	Gagliardo - Nirenberg inequality	40
4	Main hyperbolic equations and energy type estimates	43
4.1	Linear wave and Klein-Gordon equations	43
4.2	Self - adjoint generators	46
4.3	Energy estimate for Klein - Gordon equation	51
4.4	Some other hyperbolic problems of mathematical physics	54
4.5	Examples of nonlinear hyperbolic equations	57
5	Stationary phase method and pseudodifferential operators	60
5.1	Stationary phase method	60
5.2	Pseudodifferential operators	65
6	Complex interpolation and fractional Sobolev spaces on flat space	75
6.1	Abstract complex interpolation for couple of Banach spaces	75
6.2	Interpolation for sequences with values in Banach spaces	79
6.3	Interpolation for semigroups in Banach spaces	83
6.4	Fourier multipliers	87
6.5	Complex interpolation in H_p^s	90
6.6	Multiplicative inequalities in H_p^s	92
7	Weighted Sobolev spaces on flat space	96
7.1	Abstract localized norms	96
7.2	Localized Sobolev norms and weighted spaces	99
7.3	The generalized Sobolev spaces	99
7.4	The weighted Sobolev spaces $H_p^s(\rho)$	103
7.5	Sobolev spaces associated to Lie algebras	107