# Preface to the First Edition

I and H. Kozono organized seminars entitled "Applications of method in real analysis to nonlinear partial differential equations" as a Regional Workshop (RW) planned by the Mathematical Society of Japan at the department of mathematics of Tsukuba University in 1996-1997. One seminar from December 2 through 12 in 1996 was concerned with the Navier-Stokes equation. Another one from January 13 through 23 in 1997 was concerned with the nonlinear hyperbolic problem. This volume was partially presented at the nonlinear hyperbolic week. My discussion with V. Georgiev and P. D'Ancona at the summer school organized by V. Georgiev at the Bulgarian Academy of Science in 1996 became the basis of this RW seminar.

Most of the standard theorems of global in time existence for solutions of the nonlinear evolution equations in mathematical physics depend heavily upon estimates for the solution's total energy. Typically, to prove the global existence of a smooth solution, one argues that a certain amount of energy would necessarily be dissipated in the development of a singularity, which is limited by virtue of small data assumptions so far. It is well known that without small data assumption, present day analysis yields only a local in time solution in many cases, except for some semilinear evolution equations with good sign.

Under the small data assumption, the main observation is devoted to the investigation of the dissipative mechanism of linearized equations, which is described by the decay estimate of solutions mathematically. V. Georgiev is one of the most excellent mathematicians who created outstanding a priori estimates about linear hyperbolic equations in mathematical physics, which yield solutions of the corresponding nonlinear hyperbolic equations under small data assumption.

The aim of this lecture note is to explain how to derive sharp a priori estimates which enable one to prove a global in time existence of solutions to semilinear wave equation and non-linear Klein-Gordon equation by using some useful tools which include a basic fact of functional analysis, interpolation theorem, Fourier transform, Sobolev spaces, psudodifferential operator, weighted Sobolev spaces with fractional order and Fourier transform on the hyperboloid.

The core of the lecture note is §8, which is devoted to Fourier transform on manifolds with constant negative curvature. In fact, some Sobolev estimates on the hyperboloid have been successfully used by S. Klainerman and L. Hörmander in 80's and 90's to obtain suitable $L^\infty$ decay estimates for the solution of the wave and Klein-Gordon equations. The use of Fourier transform on the hyperboloid enables one to use powerful tools such as the interpolation methods and psudodifferential operator approach in getting better $L^p$ weighted a priori estimates.

I believe that this lecture note encourages motivated students who want to study a modern theory of partial differential equations and that it gives many researchers very useful tools which are applicable to solve important problems arising from mathematical physics

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