

# Contents

<b>Notations and symbols</b>	<b>xi</b>
<b>Introduction</b>	<b>1</b>
<b>1 Almost periodic functions</b>	<b>5</b>
1.1 Definition and some properties . . . . .	5
1.2 Mean values . . . . .	10
1.3 Convolutions . . . . .	13
1.4 Approximation theorem . . . . .	18
1.5 Parseval equality . . . . .	23
<b>2 Probability measure <math>\mathbf{P}</math> on <math>\mathbb{R}^{\mathbb{B}}</math></b>	<b>27</b>
2.1 Definition of the probability measure $\mathbf{P}$ . . . . .	27
2.2 Limit theorem on the probability space $(\mathbb{R}^{\mathbb{B}}, \mathbf{P})$ . . . . .	32
<b>3 Complex random variable <math>\sum_p -\log(1 - \frac{e(-\log p)}{p^\sigma})</math> on <math>(\mathbb{R}^{\mathbb{B}}, \mathbf{P})</math></b>	<b>43</b>
3.1 Complex random variables $e(\lambda)$ . . . . .	43
3.2 Logarithm functions of a complex variable . . . . .	47
3.3 Complex random variable $\sum_p -\log(1 - \frac{e(-\log p)}{p^\sigma})$ . . . . .	49
3.4 Some properties of the distribution of $\sum_p -\log(1 - \frac{e(-\log p)}{p^\sigma})$ . . . . .	51
<b>4 Riemann zeta function</b>	<b>67</b>
4.1 Euler-Maclaurin summation formula . . . . .	67
4.2 Analytic continuation to the entire complex plane . . . . .	75
4.3 Functional equation . . . . .	81
4.4 No zeros on the line $\operatorname{Re} s = 1$ . . . . .	93
<b>5 Bohr-Jessen limit theorem</b>	<b>97</b>
5.1 Log zeta function . . . . .	97
5.2 Presentation of the main theorem . . . . .	101
5.3 Proof of the main theorem . . . . .	104
<b>6 Some facts from analytic number theory</b>	<b>119</b>
6.1 Square mean value estimate of $\zeta(s)$ . . . . .	119
6.2 Stirling's formula and estimate of $\Gamma^{(l)}(\sigma + \sqrt{-1}t)$ . . . . .	148
6.3 Carlson's mean value theorem . . . . .	171