Preface

In [22], Ivrii and Petkov introduced the notion of fundamental matrix, which now called the Hamilton map, and proved that if the Cauchy problem is C^∞ well posed for any lower order term then the characteristics are at most double and at every double characteristic point the Hamilton map has non-zero real eigenvalues, now called *effectively hyperbolic*. If the Hamilton map has no non-zero real eigenvalue, that is *noneffectively hyperbolic* case, they also proved, under some restrictions, in order that the Cauchy problem is C^∞ well posed the subprincipal symbol must lie in some interval on the real line, which depends on the reference double characteristic point. This was a breakthrough in researches on hyperbolic operators with multiple characteristics (to feel the impact of that paper, see for example [14]). They conjectured that effectively hyperbolic operator is strongly hyperbolic, that is if the Hamilton map has non-zero real eigenvalues at every double characteristic then the Cauchy problem is C^∞ well posed for any lower order term. This conjecture has been proved affirmatively in [25], [29], [30], [42]. On the other hand, the necessary condition for the C^∞ well-posedness for noneffectively hyperbolic operator, mentioned above was completed by removing the restrictions in [18] and now called the Ivrii-Petkov-Hörmander condition.

If the Hamilton map verifies some spectral restrictions, the Cauchy problem for noneffectively hyperbolic operators is C^∞ well posed under the strict Ivrii-Petkov-Hörmander condition which was proved in [24], [18]. The main remaining question is thus whether this restriction on the Hamilton map can be removed or we need other necessary conditions for the C^∞ well-posedness. Much work has been devoted to this or related questions, see [41], [44], [47], [4], [6], [20].

It has been recognized that what is crucial to the C^∞ well-posedness is not only the Hamilton map but also the behavior of the Hamilton flow near the double characteristic manifold and the Hamilton map itself is not enough to determine completely the behavior of the Hamilton flow. Strikingly enough, if the Hamilton flow lands tangentially on the double characteristic manifold then the Cauchy problem is not C^∞ well posed even though we assume the Levi conditions, only well posed in the Gevrey class of order $1 \leq s < 5$ as proved in [7] (the arguments of the proof of non well-posedness there was incomplete and then we complete the proof here). On the other hand if the Hamilton flow does not touch the double characteristic manifold tangentially then the above mentioned result still holds; the Cauchy problem is C^∞ well posed under the