Preface

The theory of hypergeometric functions is a venerable subject with three centuries of history. The last twenty years have seen the emergence of the Aomoto-Gelfand multivariable theory of hypergeometric functions. There are advanced books on the subject by Aomoto and Kita [AK] and by Varchenko [V2]. A significant feature of this work is the essential use of the theory of arrangements. Arrangements of hyperplanes as a separate subject is fairly new. In the first comprehensive book on arrangements [OT1], we made only a brief reference to the connections with current work on multivariable hypergeometric functions. Since then the extent of these connections has been clarified. These notes are intended to serve as an elementary introduction to hypergeometric functions from the point of view of arrangement theory and as an exposition of the connections between these subjects. A more advanced overview is presented in the very illuminating survey by Looijenga [L]. Time and space limitations forced major omissions, most notably: systems of hypergeometric differential equations, central in Gelfand's work [Ge, GZ]; Lie theory applications and Knizhnik-Zamolodchikov equations, treated by Etingof, Frenkel, and Kirillov [EFK]; and work on root systems and hypergeometric functions by Heckman and Opdam [He1, He2, HO, O1, O2]. Classical hypergeometric functions have been particularly important in physics. The generalizations described here are also relevant to recent work in theoretical physics, especially in the representation theory of quantum groups and in conformal field theory. These topics are explored in Varchenko's book [V2].

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