Preface

The birational classification theory of higher-dimensional algebraic varieties is one of the most important subjects in algebraic geometry. We need to construct, study, and calculate various birational invariants for the classification. In the opinion of the author, these invariants related to the canonical divisor K_X of a variety X are most essential for the purpose of recovering the information vital to determine the variety X.

In dimension one, for example, the (geometric) genus $p_g(X) = \dim |K_X| + 1$ characterizes the nature of a non-singular projective curve X. In arbitrary dimension, the theory of Kodaira dimension by Iitaka [**43**], [**44**] tells us that the plurigenera $P_m(X) = \dim |mK_X| + 1$ for $m > 0$ and the Kodaira dimension $\kappa(X)$ determined by $P_m(X)$ also characterize the nature of a variety X.

The minimal model theory for higher-dimensional varieties also requires the information on the canonical divisor at the heart of its construction. For example, the following assertion gives the first step to construct minimal models: 'if K_X is not nef, then there exist an extremal ray and its contraction morphism.'

The author thinks that the canonical divisor can be compared to the 'navel' of a variety in the sense that the navel is an origin of a human body.

We list the following important conjectures arising from the study of canonical divisors:

- The canonical ring is a finitely generated algebra over the base field;
- Iitaka's conjecture $C_{n,m}$: $\kappa(X) \geq \kappa(X/Y) + \kappa(Y)$ for an algebraic fiber space $X \to Y$;
- Existence and termination of flips in the minimal model program;
- The abundance conjecture: K_X is semi-ample for a minimal model X;
- Deformation invariance of plurigenera, etc.

The study of canonical divisors has two sides: One is the study of properties valid not only for the canonical divisor but also for all the divisors in general. The other is finding some theorems, valid specifically for the divisors close to the canonical divisor, from Hodge theory, from the analytic methods for complex analytic varieties, or from the Frobenius maps of schemes of characteristic $p > 0$.

The study of the divisors in general is an old subject in algebraic geometry. The notion of linear systems originally comes from the classical projective geometry. A linear system on a variety parametrizes effective divisors linearly equivalent to