APPENDIX TO CHAPTER 4. POINTWISE LIMITS OF BAYES PROCEDURES

This appendix contains a proof of Theorem 4.14, which was used to establish the complete class Theorems 4.16 and 4.24, and will be used again in Chapter 7. As already noted, this theorem has nothing in particular to do with exponential families, but its proof is included here since it is not readily accessible elsewhere. We will state and prove it below in a convenie form which is more general than that stated in Theorem 4.14.

4A.1 Setting

Let $\{p_\theta(x)\colon \theta \in \Theta\}$ be any family of probability densities relativ to a σ-finite measure ν on a measure space $X, \mathcal{B}$. Assume

$$p_\theta(x) > 0 \qquad x \in X, \quad \theta \in \Theta \qquad . \tag{1}$$

(This assumption is actually used only in Proposition 4A.11 and Theorem 4A.12 Let the action space, A, be a closed convex subset of Euclidean space. The loss function is $L\colon \Theta \times A \to [0, \infty)$. Assume $L(\theta, \cdot)$ is a lower semicontinuous function for each $\theta \in \Theta$. Assume also that

$$\lim_{||a|| \to \infty} L(\theta, a) = \infty \quad , \qquad \theta \in \Theta \ . \tag{2}$$

(If A is a bounded set this is trivially satisfied.) If A is bounded let $A^* = A$; if A is unbounded let $A^* = A \cup \{\underset{\sim}{\iota}\}$ denote the one-point compactification of A. Extend the function $L(\theta, \cdot)$ to A^* by defining

$$L(\theta, \underset{\sim}{\iota}) = \infty \ . \tag{3}$$

A randomized decision procedure on A^* is a kernel $\delta(\cdot|\cdot)$ for which