Inequalities in Statistics and Probability
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INTRODUCTION[1]

BY Y. L. TONG
University of Nebraska

As noted by Pólya (1967), "Inequalities play a role in most branches of mathematics and have widely different applications." This is certainly true in statistics and probability. From the viewpoint of applications, inequalities have become a useful tool in estimation and hypothesis-testing problems (such as for yielding bounds on the variances of estimators and on the probability contents of confidence regions, and for establishing monotonicity properties of the power of certain tests), in multivariate analysis, in reliability theory, and so forth. Perhaps the usefulness of inequalities can be best illustrated by the following situation in reliability theory: Under certain circumstances it is desirable or necessary to determine whether or not the reliability of a system meets a given specification. The evaluation of the true reliability of a complex system is not always feasible. But if an inequality in the form of a lower bound on the system reliability can be easily obtained, and if the lower bound already meets the specification, then one knows for sure that the system meets or exceeds the specification.

On the other hand, the theory of inequalities in statistics and probability has intrinsic interest and importance and need not rely only on applications. For deriving such inequalities one usually studies a problem from several different approaches, such as monotonicity properties via concepts of stochastic ordering of random variables and distributions, positive or negative dependence and/or association properties via a mixture of distributions or monotone transformations of random variables, Schur concavity and the notion of majorization via the diversity of the components of a vector (or matrix), and so on. Thus the study of inequalities *per se* can provide a better understanding of the interrelationships among the random variables and their transformations, and may reveal new information concerning the complicated structure of probability distributions. This in turn provides new insights, ideas, and approaches for solving a variety of problems in statistics and probability.

The general study of the theory of inequalities in statistics and probability is, of course, closely related to the developments of inequalities in mathematics. As Mitrinović pointed out (1970, p. v), although "the theory of inequalities (in mathematics) began its development from (the days of) C. F. Gauss, A. L. Cauchy, and P. L. Cebysev," it is "the classical work *Inequalities* by G. H. Hardy, J. E. Littlewood and G. Pólya (1934, 1952) . . . which transformed the field of inequalities from a collection of isolated formulas into a systematic discipline." After the publication of the second editon of their book in 1952, there have been several other volumes on mathematical inequalities; such

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