FOREWORD

The subject matter of these conference proceedings comes in many guises. Some view it as the study of probability distributions with fixed marginals; those coming to the subject from probabilistic geometry see it as the study of copulas; experts in real analysis think of it as the study of doubly stochastic measures; functional analysts think of it as the study of Markov operators; and statisticians say it is the study of possible dependence relations between pairs of random variables. All are right since all these topics are isomorphic.

This diversity of viewpoints reflects a diversity of origins. Doubly stochastic measures arose, at least in part, from attempts to generalize the concept of doubly stochastic matrices to a continuous setting. Copulas were introduced because of an interest in the ways joint distribution functions are related to their marginals. Probability distributions with fixed marginals are a natural object in the study of nonparametric statistics; they also arise in problems that are not originally statistical in nature, e.g., in the determination of the optimal translocation of masses (the Monge-Kantorovich problem) as well as in the construction of probability metrics which are of increasing importance in the investigation of stability properties of stochastic models.

Over the years there has been a steady accumulation of results in all these areas, a growing realization of the fact that there are close links between them, and a gradual increase in the pace of research. For example, to anyone familiar with both definitions, it is quickly obvious that a copula is simply the joint distribution function of a doubly stochastic measure on the unit square, whence there is a one-to-one correspondence between these two concepts. In 1966 J. R. Brown showed that there is a "nice" homeomorphism between Markov operators and doubly stochastic measures on the unit square. Earlier, in 1959, A. Sklar, in response to a query by M. Fréchet, had shown that if H is a two-dimensional joint distribution function with one-dimensional margins F and G, then there exists a copula C satisfying $H(x,y) = C(F(x), G(y))$ and that the copula C is unique when the ranges of both F and G are each the entire unit interval $[0,1]$. When the copula is unique, it can be associated with the type of dependence that exists between random variables having H as their joint distribution. When the copula fails to be unique (see A. W. Marshall's paper in this volume) the situation is far more complicated and one will be forced to work with the subcopulas which are unique. In case of uniqueness, the Fréchet-Hoeffding bounds for H may be looked upon as the first probabilistic explanation of the type of dependence corresponding to certain copulas. More recently there has been a growing realization by statisticians that copulas provide a unified way of studying many nonparametric