Preface

The purpose of this book is to present a version of multivariate statistical theory in which vector space and invariance methods replace, to a large extent, more traditional multivariate methods. The book is a text. Over the past ten years, various versions have been used for graduate multivariate courses at the University of Chicago, the University of Copenhagen, and the University of Minnesota. Designed for a one year lecture course or for independent study, the book contains a full complement of problems and problem solutions.

My interest in using vector space methods in multivariate analysis was aroused by William Kruskal's success with such methods in univariate linear model theory. In the late 1960s, I had the privilege of teaching from Kruskal's lecture notes where a coordinate free (vector space) approach to univariate analysis of variance was developed. (Unfortunately, Kruskal's notes have not been published.) This approach provided an elegant unification of linear model theory together with many useful geometric insights. In addition, I found the pedagogical advantages of the approach far outweighed the extra effort needed to develop the vector space machinery. Extending the vector space approach to multivariate situations became a goal, which is realized here. Basic material on vector spaces, random vectors, the normal distribution, and linear models take up most of the first half of this book.

Invariance (group theoretic) arguments have long been an important research tool in multivariate analysis as well as in other areas of statistics. In fact, invariance considerations shed light on most multivariate hypothesis testing, estimation, and distribution theory problems. When coupled with vector space methods, invariance provides an important complement to the traditional distribution theory-likelihood approach to multivariate analysis. Applications of invariance to multivariate problems occur throughout the second half of this book.

A brief summary of the contents and flavor of the ten chapters herein follows. In Chapter 1, the elements of vector space theory are presented. Since my approach to the subject is geometric rather than algebraic, there is an emphasis on inner product spaces where the notions of length, angle, and orthogonal projection make sense. Geometric topics of particular importance in multivariate analysis include singular value decompositions and angles between subspaces. Random vectors taking values in inner product spaces is the general topic of Chapter 2. Here, induced distributions, means, covariances, and independence are introduced in the inner product space setting. These results are then used to establish many traditional properties of the multivariate normal distribution in Chapter 3. In Chapter 4, a theory of linear models is given that applies directly to multivariate problems. This development, suggested by Kruskal's treatment of univariate linear models, contains results that identify all the linear models to which the Gauss–Markov Theorem applies.

Chapter 5 contains some standard matrix factorizations and some elementary Jacobians that are used in later chapters. In Chapter 6, the theory of invariant integrals (measures) is outlined. The many examples here were chosen to illustrate the theory and prepare the reader for the statistical applications to follow. A host of statistical applications of invariance, ranging from the invariance of likelihood methods to the use of invariance in deriving distributions and establishing inde-