DECIDABILITY QUESTIONS FOR THEORIES OF MODULES

F. POINT

§0. Introduction.

Which finite rings with identity have a decidable theory of unitary left modules? This question has been raised by S. Burris and R. McKenzie in their paper on decidable varieties with modular congruence lattices. They showed that if a locally finite variety with modular congruence lattice does not decompose as a product of a discriminator variety and an affine variety, then it interprets the theory of all finite graphs. Then, they reduced the problem of classifying the decidable locally finite affine varieties to the problem of classifying the finite rings which have a decidable theory of modules.

First, we will see how this question arises in the context of decidable locally finite varieties. Then, we will restrict our attention to the decidability question for theories of modules. We will establish a connection between the decidability of the theory of modules over a finite-dimensional algebra and the representation type of that algebra.

This leads to the following questions: what are the relationships

- between the theory of R-modules and the theory of finitely generated R-modules?
- between theories of modules which are Morita equivalent?

§1. Locally finite varieties.

A *variety* is a class of L-structures, where the language L only contains function symbols, defined by some set of equations (or equivalently closed under products, substructures and homomorphisms). A variety is *locally finite* if every finitely generated algebra is finite.

S. Burris and R. McKenzie proved a decomposition theorem for decidable locally finite varieties with modular congruence lattice. They show that it decomposes as the product of a discriminator variety and an affine variety. (See [B,M]).

R. McKenzie and M. Valeriote generalized their decomposition theorem for decidable locally finite varieties. Before stating the result of McKenzie and Valeriote, we make this notion of decomposition precise.