

SENSE AND DENOTATION AS ALGORITHM AND VALUE

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§1. Introduction.

In his classic 1892 paper *On sense and denotation* [12], Frege first contends that in addition to their **denotation** (*reference*, *Bedeutung*), proper names also have a **sense** (*Sinn*) “wherein the mode of presentation [of the denotation] is contained.” Here proper names include common nouns like “the earth” or “Odysseus” and descriptive phrases like “the point of intersection of lines L_1 and L_2 ” which are expected by their grammatical form to name some object. By the second fundamental doctrine introduced by Frege in the same paper, they also include declarative sentences: “[a simple, assertoric sentence is] to be regarded as a proper name, and its denotation, if it has one, is either the True or the False.” Thus every sentence *denotes* (or *refers to*) *its truth value* and *expresses its sense*, which is “a mode of presentation” of its truth value: this is all there is to the meaning of a sentence as far as logic is concerned. Finally, Frege claims that although sense and denotation are related (the first determines the second), they obey separate *principles of compositionality*, so that the truth value of a complex sentence ϕ is determined solely by the denotations of its constituent parts (terms and sentences), whatever the senses of these constituents parts may be. This is the basic principle which has made possible the development of sense-independent (two-valued, classical) *denotational semantics* for predicate logic, a variety of richer formal languages and (at least) fragments of natural language.

Consider, for example, the arithmetical sentences

$$\theta \equiv 1 + 0 = 1, \tag{1}$$

$$\eta \equiv \text{there exist infinitely many primes}, \tag{2}$$

$$\rho \equiv \text{there exist infinitely many twin primes}. \tag{3}$$

The first two are true, the third is a famous open problem. Surely we understand all three and we understand them differently, even as we recognize the first two to be true and concede that we do not know the truth value of the third. Frege would say that they mean different things because they express different senses, but he did not define sense: what exactly *is* the sense of θ and how does it

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