CHAPTER 1

GENERALITIES ON ABSTRACT LOGICS AND SENTENTIAL LOGICS

In this chapter we include the main definitions, notations, and general properties concerning logical matrices, abstract logics and sentential logics. Most of the results reproduced here are not new; however, those concerning abstract logics are not well-known, so it seems useful to recall them in some detail, and to prove some of the ones that are new. Useful references on these topics are Brown and Suszko [1973], Burris and Sankappanavar [1981] and Wójcicki [1988].

Algebras

In this monograph (except in Chapter 5, where we deal with examples) we will always work with algebras $\boldsymbol{A} = \langle A, \dots \rangle$ of the same, arbitrary, similarity type; thus, when we say "every/any/some algebra" we mean "of the same type". By $\mathrm{Hom}(\boldsymbol{A}, \boldsymbol{B})$ we denote the set of all ***homomorphisms*** from the algebra $\boldsymbol{A}$ into the algebra $\boldsymbol{B}$. The set of ***congruences*** of the algebra $\boldsymbol{A}$ will be denoted by $\mathrm{Con}\,\boldsymbol{A}$. Many of the sets we will consider have the structure of a (often complete, or even algebraic) lattice, but we will not use a different symbol for the lattice and for the underlying set, since no confusion is likely to arise. Given any class K of algebras, the set $\mathrm{Con}_{\mathsf{K}}\boldsymbol{A} = \{\theta \in \mathrm{Con}\,\boldsymbol{A} : \boldsymbol{A}/\theta \in \mathsf{K}\}$ is called the set of K***-congruences*** of $\boldsymbol{A}$; while this set is ordered under $\subseteq$, in general it is not a lattice. This set will play an important role in this monograph.

Formulas, equations, interpretations

We will denote by $\boldsymbol{Fm} = \langle Fm, \dots \rangle$ the absolutely free algebra of the similarity type under consideration generated by some fixed but unspecified set Var, which we assume to be countably infinite. $\boldsymbol{Fm}$ is usually called the ***algebra of formulas*** (or the algebra of terms), and the elements of Var the ***variables***, or atomic formulas. The letters $p, q, \dots$ will denote variables, and the formulas will be denoted by lowercase Greek letters such as $\varphi, \psi, \xi, \eta, \dots$, while uppercase