

# 1. Definitions and Preliminaries

A major part of this chapter serves to review and fix notation and terminology. The material is standard. Readers familiar with the notions addressed might therefore only want to refer back to particular definitions at later points. The main issues of the individual sections are the following:

- Section 1.1 sums up the basics about structures, global relations, logics and types that are relevant for our purposes.
- In Section 1.2 we consider algorithms that deal with structures as inputs and fix some corresponding conventions. Recognizability of classes of finite structures and computability of global relations are discussed.
- The bounded variable fragments of infinitary logic, and the fixed-point logics, are presented in Section 1.3. We also provide some typical examples for the expressive power of these logics.
- Section 1.4 contains some preliminary material about types and definability in the relevant fragments of infinitary logic.
- Section 1.5 deals with interpretations, a concept that plays an important rôle in many definability considerations.
- In Section 1.6 we review the notions of generalized quantifiers and Lindström extensions. In particular we define the class of cardinality Lindström quantifiers.
- Section 1.7 fixes some terminology with respect to the notion of canonization and of complete invariants for arbitrary equivalence relations. We also sketch some technicalities and conventions concerning orderings and pre-orderings.

## 1.1 Structures and Types

### 1.1.1 Structures

We deal with *finite structures* exclusively.  $\text{fin}[\tau]$  is the class of all finite  $\tau$ -structures. Unless explicitly stated otherwise,  $\tau$  stands for some finite and