# Lambek Calculus and Formal Languages

(Extended abstract)

Mati Pentus

Department of Mathematical Logic
Faculty of Mechanics and Mathematics
Moscow State University
Moscow, Russia, 119899
pentus@lpcs.math.msu.ru

## Introduction

The systematic study of generating grammars was started by N. Chomsky in the 50s (cf. [10]). He defined several classes of generating grammars, which are interesting for both linguists and mathematicians, e. g. context-sensitive grammars, context-free grammars, and linear grammars. On the other hand, categorial grammars were studied by Y. Bar-Hillel, J. Lambek and others. The notion of a basic categorial grammar was introduced in [1]. In the same paper it was proved that the languages recognized by basic categorial grammars are precisely the context-free ones.

Another kind of categorial grammar was introduced by J. Lambek [15]. These grammars are based on a syntactic calculus, presently known as the Lambek calculus. Chomsky [11] conjectured that these grammars are also equivalent to context-free ones. In [12] Cohen proved that every basic categorial grammar (and, thus, every context-free grammar) is equivalent to a Lambek grammar. He also proposed a proof of the converse. However, as pointed out in [6], this proof contains an error. Buszkowski proved that some special kinds of Lambek grammars are context-free [6, 8, 9]. These grammars use weakly unidirectional types or types of order at most two.

The first result of this paper (Theorem 1) says that Lambek grammars generate only context-free languages. Thus they are equivalent to context-free grammars and also to basic categorial grammars. This fact (sometimes called *the Chomsky Conjecture*) was proved in [16] and [17].

The inteneded syntactic string models, i.e., *free semigroup models* (also called *language models* or *L-models*) for the Lambek calculus were considered in [3], [4], and [5]. The more general class of *groupoid models* has been studied in [7], [13], and [14]. In [4] W. Buszkowski established that the product-free fragment of the Lambek calculus is L-complete (i.e., complete w.r.t. free semigroup models), using the canonical model. The question of L-completeness of the full Lambek calculus remained open (cf. [2]).

The second result of this paper (Theorem 2) gives a positive answer to this question. The proof has been publised in [18] and [19].