Beyond Gödel's Theorem: Turing Nonrigidity Revisited

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In [4] it was argued that incomputability in nature dictates a mathematical model (due to Turing [39]) based on mechanical processes relative to appropriate abstractions of incomputable phenomena. (One should refer to that paper for background historical and technical detail.) For instance, it is Turing definability rather than the more familiar notions of provability, and completeness of axiomatic theories, which are more relevant to an analysis of the scope of scientific understanding in the real world; while recent results concerning Turing invariance and nonrigidity have both negative and positive consequences for science as a means to knowledge. Turing nonrigidity (see [6]) may reinforce scepticism about a narrow perspective based on scientific observation, as modelled by Turing computable processes: but the proliferation of invariant substructures of the Turing universe (see, for example, Cooper [5], Nies, Shore and Slaman [27] or Odifreddi [28]) can be viewed as reflecting negatively on the more radical postmodernist and (post-) structuralist views of the roles of culture and language in relation to science (cf. Gross and Levitt [17]) – objective reality does exist. Such comments can be framed in terms of qualifications to the Duhem-Quine thesis ("Any statement can be held true come what may, if we make drastic enough adjustments elsewhere in the system.", Quine [33], p. 43), and have obvious consequences for related empiricist and pragmatist views of the world (cf. again Quine, p. 44, "Physical objects are conceptually imported into the situation as convenient intermediaries – not by definition in terms of experience, but simply as irreducible points comparable, epistemologically, to the gods of Homer.") Of course, the theory itself does indicate difficulties in substantiating the Turing model, but, if not overstretched (viz. the ubiquitous Gödel's [15], [16] Theorem) such asymptotic representations can be useful and productive adjuncts to subjective intuition. For instance, unlike in mathematics where

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