

# Intervals Without Critical Triples

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**Abstract.** This paper is concerned with the construction of intervals of computably enumerable degrees in which the lattice  $M_5$  (see Figure 1) cannot be embedded. Actually, we construct intervals  $\mathcal{I}$  of computably enumerable degrees without any weak critical triples (this implies that  $M_5$  cannot be embedded in  $\mathcal{I}$ , see Section 2). Our strongest result is that there is a  $\text{low}_2$  computably enumerable degree  $\mathbf{e}$  such that there are no weak critical triples in either of the intervals  $[\mathbf{0}, \mathbf{e}]$  or  $[\mathbf{e}, \mathbf{0}']$ .

## 1 Introduction

A set of natural numbers is computably (or recursively) enumerable if it is the range of a function computed by a Turing machine. We say one set of natural numbers,  $A$ , is Turing computable from another,  $B$ , if there is a Turing machine, which using an oracle for  $B$ , computes  $A$ . Equivalence classes under this

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