Intervals Without Critical Triples

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Abstract. This paper is concerned with the construction of intervals of computably enumerable degrees in which the lattice M_5 (see Figure 1) cannot be embedded. Actually, we construct intervals $\mathcal{I}$ of computably enumerable degrees without any weak critical triples (this implies that M_5 cannot be embedded in $\mathcal{I}$, see Section 2) . Our strongest result is that there is a low$_2$ computably enumerable degree e such that there are no weak critical triples in either of the intervals $[\mathbf{0}, \mathbf{e}]$ or $[\mathbf{e}, \mathbf{0}']$.

1 Introduction

A set of natural numbers is computably (or recursively) enumerable if it is the range of a function computed by a Turing machine. We say one set of natural numbers, A, is Turing computable from another, B, if there is a Turing machine, which using an oracle for B, computes A. Equivalence classes under this