Proof. By 17.5. □

A maximal set is a coinfinite RE set A such that for every coinfinite RE set B including A, $B - A$ is finite. Thus a maximal set is a coinfinite RE set with as few RE sets as possible including it.

It is fairly easy to show that a maximal set is hypersimple. However, it is not a simple matter to show that maximal sets exist; this was done by Friedberg. The final result of a series of investigations of this question is the following theorem of Martin: an RE degree **a** contains a maximal set iff **a**' = **0**''. Thus this notion of largeness does tell us more about the degree than our previous notions, but does not tell us that the degree cannot be **0**'.

18. Function of Reals

We now extend our notion of a function to allow reals as arguments. (We could allow all total functions as arguments; but this would complicate matters without really adding anything, since a function can be replaced by its contraction.) We use lower case Greek letters, usually α, β, and γ, for reals. When the value of m is not important, we write $\vec{\alpha}$ for $\alpha_1,\ldots,\alpha_m$. We use $\mathbb{R}$ for the class of reals and $\mathbb{R}^{m,k}$ for the class of all $(m+k)$–tuples $(\alpha_1,\ldots,\alpha_m,x_1,\ldots,x_k)$. An (m,k)–ary function is a mapping of a subset of $\mathbb{R}^{m,k}$ into ω. (Thus a $(0,k)$–ary function is just a k–ary function.) From now on, a function is always an (m,k)–ary function for some m and k. Such a function is total if its domain is all of $\mathbb{R}^{m,k}$. An (m,k)–ary relation is a subset of $\mathbb{R}^{m,k}$. We define the representing function of such a relation as before.

Note that the real arguments to a function or relation must precede the number arguments. It may sometimes be convenient to write them in a different order. It is then understood that we are to move all real arguments to the left of all number arguments without otherwise changing the order of the arguments.

Now we consider how to extend the idea of computability. The new