$$z = \overline{\chi_P}(y) \leftrightarrow Seq(z) \mathbin{\&} lh(z) = y \mathbin{\&} (\forall i < y)((z)_i = \chi_P(i)).$$

Hence by the table, it will suffice to show that $w = \chi_P(i)$ is Σ^0_{n+1}. Since P is Π^0_n, this follows from

$$w = \chi_P(i) \leftrightarrow (w = 1 \mathbin{\&} P(i)) \vee (w = 0 \mathbin{\&} \neg P(i))$$

and the table. □

14.9. COROLLARY. A relation is Δ^0_{n+1} iff it is recursive in Π^0_n.

Proof. A relation R is Δ^0_{n+1} iff both R and $\neg R$ are Σ^0_{n+1}; hence, by Post's Theorem, iff both R and $\neg R$ are RE in Π^0_n. By the relativized version of 14.6, this holds iff R is recursive in Π^0_n. □

Since $\neg R$ is recursive in R and $R = \neg\neg R$ is recursive in $\neg R$, 12.4 and the table show that we can replace Π^0_n by Σ^0_n in both Post's Theorem and its corollary.

15. Degrees

If F and G are total functions, we let $F \leq_R G$ mean that F is recursive in G. By 12.5,

$$(1) \qquad F \leq_R F;$$

and by the Transitivity Theorem

$$(2) \qquad F \leq_R G \mathbin{\&} G \leq_R H \rightarrow F \leq_R H.$$

Let $F \equiv_R G$ mean $F \leq_R G \mathbin{\&} G \leq_R F$. It follows from (1) and (2) that $\equiv_R$ is an equivalence relation. The equivalence class of F is called the degree of F and is designated by dg F. By a degree, we mean the degree of some total function. We use small boldface letters, usually **a**, **b**, **c**, and **d**, for degrees.

We let $\mathrm{dg}(F) \leq \mathrm{dg}(G)$ if $F \leq_R G$. By (2), this depends only on dg(F) and dg(G), not on the choice of F and G in these equivalence classes. It follows from (1) and (2) that $\leq$ is a partial ordering of the degrees, i.e., that

$$\mathbf{a} \leq \mathbf{a},$$

$$\mathbf{a} \leq \mathbf{b} \mathbin{\&} \mathbf{b} \leq \mathbf{a} \rightarrow \mathbf{a} = \mathbf{b},$$