$$L(y,x,f) \simeq G(x) \qquad \text{if } y = 0,$$
$$\simeq H(\{f\}(y \dot{-} 1, 2x), y \dot{-} 1, x) \qquad \text{otherwise.}$$

Then we use the Recursion Theorem to obtain a recursive $F$ with an index $f$ such that $F(y,x) \simeq L(y,x,f)$. Clearly $F$ satisfies (2); so the function defined by (2) is recursive.

## 9. Church's Thesis

We have already remarked that it is clear that every recursive function is computable. The statement that every computable function is recursive is known as Church's Thesis. It was proposed by Church about 1934 and has since come to be accepted by almost all logicians. We shall discuss the reasons for this.

Since the notion of a computable function has not been defined precisely, it may seem that it is impossible to give a proof of Church's Thesis. However, this is not necessarily the case. We understand the notion of a computable function well enough to make some statements about it. In other words, we can write down some axioms about computable functions which most people would agree are evidently true. It might be possible to prove Church's Thesis from such axioms. However, despite strenuous efforts, no one has succeeded in doing this (although some interesting partial results have been obtained).

We are thus reduced to trying to give arguments for Church's Thesis which seem to be convincing. We shall briefly examine these arguments.

The first argument is that all the computable functions which have been produced have been shown to be recursive, using, for the most part, the techniques which we have already described. Moreover, all the known techniques for producing new computable functions from old ones (such as definition by induction or by cases) have been shown to lead from recursive functions to recursive functions.