functions that we use will be the fact that it is recursively closed. This will enable us to prove in §9 that the class of recursive functions is the smallest recursively closed class.

## 6. Definitions of Recursive Functions

We are now going to show that certain kind of definitions of functions and relations always lead to recursive functions and relations. The simplest kind of definition of a function has the form $F(\vec{x}) \simeq$ ____, where ____ is an expression which, if defined, represents a number and which contains only previously defined symbols and variables from the sequence $\vec{x}$. Such a definition is called an <u>explicit</u> definition of $F$ <u>in</u> <u>terms</u> <u>of</u> the symbols which appear in ____ .

6.1. PROPOSITION. If $F$ is defined explicitly in terms of variables and names of recursive functions, then $F$ is recursive.

*Proof.* We suppose that $F$ is defined by $F(\vec{x}) \simeq$ ____ and use induction on the number of symbols in ____ . If ____ consists of just an $x_i$, then $F$ is an $I_i^k$ and hence is recursive. Otherwise, ____ is $G(X_1,\ldots,X_n)$ where $G$ is recursive. By the induction hypothesis, we may define a recursive function $H_i$ by $H_i(\vec{x}) \simeq X_i$. Then

$$F(\vec{x}) \simeq G(H_1(\vec{x}),\ldots,H_n(\vec{x}));$$

so $F$ is recursive by 5.2. □

The simplest type of definition of a relation has the form $R(\vec{x}) \leftrightarrow$ ____ where ____ is a statement containing only previously defined symbols and variables from the sequence $\vec{x}$. In order to make sure that this defines a relation, we insist that ____ be defined for all values of $\vec{x}$. We call such a definition an <u>explicit</u> definition of $R$ <u>in</u> <u>terms</u> <u>of</u> whatever symbols appear in ____.

6.2. PROPOSITION. If $R$ is defined explicitly in terms of variables and names of recursive functions and relations, then $R$ is recursive.

*Proof.* The definition must be $R(\vec{x}) \leftrightarrow Q(X_1,\ldots,X_n)$, where $Q$ is a