A 1–ary relation is simply a set of numbers. We understand set to mean set of numbers; we will use the word class for other kinds of sets. We use A and B for sets.

If R is a k–ary relation, the representing function of R, designated by χ_R, is the k–ary total function defined by

$$\begin{aligned} \chi_R(\vec{x}) &= 0 \qquad \text{if } R(\vec{x}), \\ &= 1 \qquad \text{otherwise.} \end{aligned}$$

A relation R is computable if the function χ_R is computable. We adopt the convention that whenever we attribute to a relation some property usually attributed to a function, we are actually attributing that property to the representing function of the relation.

3. The Basic Machine

To define our class of functions, we introduce a computing machine called the basic machine. It is an idealized machine in that it has infinitely much memory and never makes a mistake. Except for these features, it is about as simple as a computing machine can be.

For each number i, the computing machine has a register $\mathfrak{X}i$. At each moment, $\mathfrak{X}i$ contains a number; this number (which has nothing to do with the number i) may change as the computation proceeds.

The machine also has a program holder. During a computation, the program holder contains a program, which is a finite sequence of instructions. If N is the number of instructions in the program, the instructions are numbered 0, 1, ..., N–1 (in the order in which they appear in the program). The machine also has a counter, which at each moment contains a number.

To use the machine, we insert a program in the program holder; put any desired numbers in the registers; and start the machine. This causes 0 to be inserted in the counter. The machine then begins executing instructions. At